CHAPTER 22
MISCELLANEOUS (Sets, Relations, Statistics & Mathematical Reasoning)

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. A variable takes value x with frequency ${}^{n+x-1}C_x$, $x = 0, 1, 2, \ldots n$. The mode of the variable is................. *(1982 - 2 Marks)*

B True / False

1. For real numbers x and y, we write $x * y$ if $x - y + \sqrt{2}$ is an irrational number. Then, the relation* is an equivalence relation. *(1981 - 2 Marks)*

C MCQs with One Correct Answer

1. If X and Y are two sets, then $X \cap (X \cup Y)^c$ equals. *(1979)*
   (a) X (b) Y
   (c) ϕ (d) None of these.
2. The expression $\dfrac{12}{3+\sqrt{5}+2\sqrt{2}}$ is equal to *(1980)*
   (a) $1-\sqrt{5}+\sqrt{2}+\sqrt{10}$ (b) $1+\sqrt{5}+\sqrt{2}-\sqrt{10}$
   (c) $1+\sqrt{5}-\sqrt{2}+\sqrt{10}$ (d) $1-\sqrt{5}-\sqrt{2}+\sqrt{10}$
3. Select the correct alternative in each of the following. Indicate your choice by the appropriate letter only.
   Let S be the standard deviation of n observations. Each of the n observations is multiplied by a constant c. Then the standard deviation of the resulting number is *(1980)*
   (a) s (b) $c\,s$
   (c) $s\sqrt{c}$ (d) none of these
4. The standard deviation of 17 numbers is zero. Then *(1980)*
   (a) the numbers are in geometric progression with common ratio not equal to one.
   (b) eight numbers are positive, eight are negative and one is zero.
   (c) either (a) or (b) (d) none of these
5. Consider any set of 201 observations $x1, x2, \ldots. x200, x201$. It is given that $x1 < x2 < \ldots. < x200 < x201$. Then the mean deviation of this set of observations about a point k is minimum when k equals *(1981 - 2 Marks)*
   (a) $(x_1 + x_2 + \ldots + x_{200} + x_{201})/201$
   (b) x_1
   (c) x_{101}
   (d) x_{201}
6. If $x1, x2, \ldots\ldots\ldots, xn$ are any real numbers and n is any postive integer, then *(1982 - 2 Marks)*
   (a) $n\sum_{i=1}^{n} x_i^2 < \left(\sum_{i=1}^{n} x_i\right)^2$ (b) $\sum_{i=1}^{n} x_i^2 \geq \left(\sum_{i=1}^{n} x_i\right)^2$
   (c) $\sum_{i=1}^{n} x_i^2 \geq n\left(\sum_{i=1}^{n} x_i\right)^2$ (d) none of these
7. Let $S = \{1, 2, 3, 4\}$. The total number of unordered pairs of disjoint subsets of S is equal to *(2010)*
   (a) 25 (b) 34 (c) 42 (d) 41
8. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then *(2011)*
   (a) $P \subset Q$ and $Q - P \neq \varnothing$ (b) $Q \not\subset P$
   (c) $P \not\subset Q$ (d) $P = Q$

D MCQs with One or More than One Correct

1. In a college of 300 students every student reads 5 newspapers and every newspaper is read by 60 students. The number of newpapers is *(1998 - 2 Marks)*
   (a) at least 30 (b) at most 20
   (c) exactly 25 (d) none of these
2. Let $S_1, S_2, \ldots\ldots$ be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 cm, then for which of the following values of n is the area of S_n less than 1 sq. cm? *(1999 - 3 Marks)*
   (a) 7 (b) 8 (c) 9 (d) 10

E Subjective Problems

1. An investigator interviewed 100 students to determine their preferences for the three drinks : milk (M), coffee (C) and tea (T). He reported the following : 10 students had all the three drinks M, C and T; 20 had M and C; 30 had C and T; 25 had M and T; 12 had M only; 5 had C only; and 8 had T only. Using a Venn diagram find how many did not take any of the three drinks. *(1978)*

2. (a) Construct a triangle with base 9 cm and altitude 4 cm, the ratio of the other two sides being 2 : 1.

(b) Construct a triangle in which the sum of the three sides is 15 cm with base angles 60° and 45°. Justify your steps. ***(1979)***

3. A tent is made in the form of a frustrum A of a right circular cone surmounted by another right circular cone B. The diameter of the ends of the frustrum A are 8 m and 4 m, its height is 3 m and the height of the cone B is 2 m. Find the area of the canvas required. ***(1979)***

4. In calculating the mean and variance of 10 readings, a student wrongly used the figure 52 for the correct figure of 25. He obtained the mean and variance as 45.0 and 16.0 respectively. Determine the correct mean and variance. ***(1979)***

5. The diameter PQ of a semicircle is 6 cm. Construct a square $ABCD$ with points A, B on the circumference, and the side CD on the diameter PQ. Describe briefly the method of construction. ***(1980)***

6. C and D are any two points on the same side of a line L. Show how to find a point P on the line L such that PC and PD are equally inclined to the line L. Justify your steps. ***(1980)***

7. (i) Set A has 3 elements, and set B has 6 elements. What can be the minimum number of elements in the set $A \cup B$? ***(1980)***

(ii) P, Q, R are subsets of a set A. Is the following equality true ?
$R \times (P^c \cup Q^c)^c = (R \times P) \cap (R \times Q)$? ***(1980)***

(iii) For any two subset X and Y of a set A define
$X \circ Y = (X^c \cap Y) \cup (X \cap Y^c)$
Then for any three subsets X, Y and Z of the set A, is the following equality true.
$(X \circ Y) \circ Z = X \circ (Y \circ Z)$? ***(1980)***

8. Suppose $A_1, A_2, \ldots\ldots A_{30}$ are thirty sets each with five elements and $B_1, B_2, \ldots\ldots B_n$ are n sets each with three elements. Let $\overset{30}{\underset{i=1}{U}} A_i = \overset{n}{\underset{j=1}{U}} B_j = S$. Assume that each element of S belongs to exactly ten of the Ai' s and to exactly nine of the Bj' s. Find n. ***(1981 - 2 Marks)***

9. The mean square deviations of a set of observations $x_1, x_2, \ldots, x_n$ about a points c is defined to be $\frac{1}{n}\sum_{i=1}^{n}(x_1 - c)^2$.

The mean suqare deviations about –1 and +1 of a set of observatons are 7 and 3 respectively. Find the standard deviation of this set of observations. ***(1981 - 2 Marks)***

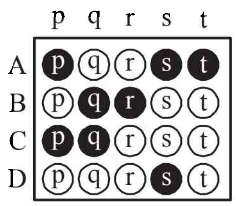
10. The marks obtained by 40 students are grouped in a frequency table in class intervals of 10 maks each. The mean and the variance obtained from this distribution are found to be 40 and 49 respectively. It was later discovered that two observations belonging to the class interval (21–30) were included in the class interval (31–40) by mistake. Find the mean and the variance after correcting the error. ***(1982 - 3 Marks)***

11. A relation R on the set of complex numbers is defined by $z_1\ R\ z_2$ if and only if $\frac{z_1 - z_2}{z_1 + z_2}$ is real. Show that R is an equivalence relation. ***(1982 - 2 Marks)***

F Match the Following

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. Match the statements given in Column-I with the intervals/union of intervals given in Column-II. ***(2011)***

Column-I

(A) The set $\{\text{Re}\left(\frac{2iz}{1-z^2}\right) : z \text{ is a complex number}, |z| = 1\ z \neq \pm 1\}$ is

(B) The domain of the function $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is

(C) If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\left\{f(\theta) : 0 \le \theta < \frac{\pi}{2}\right\}$ is

(D) If $f(x) = x^{3/2}(3x - 10), x \ge 0$ then $f(x)$ is increasing in

Column-II

(p) $(-\infty, -1) \cup (1, \infty)$

(q) $(-\infty, -0) \cup (0, \infty)$

(r) $[2, \infty)$

(s) $(-\infty, -1] \cup [1, \infty)$

(t) $(-\infty, 0] \cup [2, \infty)$

I Integer Value Correct Type

1. The value of $6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots}}}\right)$ is *(2012)*

Section-B JEE Main / AIEEE

1. In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average of the girls? **[2002]**
(a) 73 (b) 65 (c) 68 (d) 74

2. The sum of two forces is 18 N and resultant whose direction is at right angles to the smaller force is 12 N. The magnitude of the two forces are **[2002]**
(a) 13, 5 (b) 12, 6 (c) 14, 4 (d) 11, 7

3. A bead of weight w can slide on smooth circular wire in a vertical plane. The bead is attached by a light thread to the highest point of the wire and in equilibrium, the thread is taut and make an angle θ with the vertical then tension of the thread and reaction of the wire on the bead are
(a) $T = w\cos\theta$ $R = w\tan\theta$ **[2002]**
(b) $T = 2w\cos\theta$ $R = w$
(c) $T = w$ $R = w\sin\theta$
(d) $T = w\sin\theta$ $R = w\cot\theta$

4. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set **[2003]**
(a) remains the same as that of the original set
(b) is increased by 2
(c) is decreased by 2
(d) is two times the original median.

5. A couple is of moment $\vec{G}$ and the force forming the couple is $\vec{P}$. If $\vec{P}$ is turned through a right angle the moment of the couple thus formed is $\vec{H}$. If instead, the force $\vec{P}$ are turned through an angle α, then the moment of couple becomes **[2003]**
(a) $\vec{H}\sin\alpha - \vec{G}\cos\alpha$ (b) $\vec{G}\sin\alpha - \vec{H}\cos\alpha$
(c) $\vec{H}\sin\alpha + \vec{G}\cos\alpha$ (d) $\vec{G}\sin\alpha + \vec{H}\cos\alpha$.

6. The resultant of forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled then $\vec{R}$ is doubled. If the direction of $\vec{Q}$ is reversed, then $\vec{R}$ is again doubled. Then $P^2 : Q^2 : R^2$ is **[2003]**
(a) 2 : 3 : 1 (b) 3 : 1 : 1 (c) 2 : 3 : 2 (d) 1 : 2 : 3

7. A body travels a distance s in t seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r. The value of t is given by
(a) $\sqrt{2s\left(\frac{1}{f}+\frac{1}{r}\right)}$ (b) $2s\left(\frac{1}{f}+\frac{1}{r}\right)$ **[2003]**
(c) $\frac{2s}{\frac{1}{f}+\frac{1}{r}}$ (d) $\sqrt{2s(f+r)}$

8. Two stones are projected from the top of a cliff h metres high, with the same speed u, so as to hit the ground at the same spot. If one of the stones is projected horizontally and the other is projected horizontally and the other is projected at an angle θ to the horizontal then $\tan\theta$ equals **[2003]**
(a) $u\sqrt{\frac{2}{gh}}$ (b) $\sqrt{\frac{2u}{gh}}$ (c) $2g\sqrt{\frac{u}{h}}$ (d) $2h\sqrt{\frac{u}{g}}$

9. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity $\vec{u}$ and the other from rest with uniform acceleration $\vec{f}$. Let α be the angle between their directions of motion. The relative velocity of the second particle w.r.t. the first is least after a time **[2003]**
(a) $\frac{u\cos\alpha}{f}$ (b) $\frac{u\sin\alpha}{f}$ (c) $\frac{f\cos\alpha}{u}$ (d) $u\sin\alpha$

10. The upper $\frac{3}{4}$th portion of a vertical pole subtends an angle $\tan^{-1}\frac{3}{5}$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is **[2003]**
(a) $80\,m$ (b) $20\,m$ (c) $40\,m$ (d) $60\,m$.

11. Let R_1 and R_2 respectively be the maximum ranges up and down an inclined plane and R be the maximum range on the horizontal plane. Then R_1, R, R_2 are in **[2003]**
(a) H.P (b) A.G.P (c) A.P (d) G.P.

12. In an experiment with 15 observations on x, the following results were available : **[2003]**

$\Sigma x^2 = 2830,\ \Sigma x = 170$

One observation that was 20 was found to be wrong and was replaced by the correct value 30. The corrected variance is **[2003]**

(a) 8.33 (b) 78.00
(c) 188.66 (d) 177.33

13. Let R = $\{(1,3),(4,2),(2,4),(2,3),(3,1)\}$ be a relation on the set $A = \{1,2,3,4\}$.. The relation R is **[2004]**

(a) reflexive (b) transitive
(c) not symmetric (d) a function

14. Consider the following statements :
(A) Mode can be computed from histogram
(B) Median is not independent of change of scale
(C) Variance is independent of change of origin and scale.
Which of these is / are correct ? **[2004]**

(a) (A), (B) and (C) (b) only (B)
(c) only (A) and (B) (d) only (A)

15. In a series of 2 n observations, half of them equal a and remaining half equal $-a$. If the standard deviation of the observations is 2, then $|a|$ equals. **[2004]**

(a) $\frac{\sqrt{2}}{n}$ (b) $\sqrt{2}$
(c) 2 (d) $\frac{1}{n}$

16. With two forces acting at point, the maximum affect is obtained when their resultant is 4N. If they act at right angles, then their resultant is 3N. Then the forces are **[2004]**

(a) $\left(2+\frac{1}{2}\sqrt{3}\right)N$ and $\left(2-\frac{1}{2}\sqrt{3}\right)N$
(b) $\left(2+\sqrt{3}\right)N$ and $\left(2-\sqrt{3}\right)N$
(c) $\left(2+\frac{1}{2}\sqrt{2}\right)N$ and $\left(2-\frac{1}{2}\sqrt{2}\right)N$
(d) $\left(2+\sqrt{2}\right)N$ and $\left(2-\sqrt{2}\right)N$

17. In a right angle ΔABC, $\angle A = 90°$ and sides a, b, c are respectively, 5 cm, 4 cm and 3 cm. If a force $\vec{F}$ has moments 0, 9 and 16 in N cm. units respectively about vertices A, B and C, then magnitude of $\vec{F}$ is **[2004]**

(a) 9 (b) 4
(c) 5 (d) 3

18. Three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ acting along IA, IB and IC, where I is the incentre of a ΔABC are in equilibrium. Then $\vec{P} : \vec{Q} : \vec{R}$ is **[2004]**

(a) $\operatorname{cosec}\frac{A}{2} : \operatorname{cosec}\frac{B}{2} : \operatorname{cosec}\frac{C}{2}$
(b) $\sin\frac{A}{2} : \sin\frac{B}{2} : \sin\frac{C}{2}$
(c) $\sec\frac{A}{2} : \sec\frac{B}{2} : \sec\frac{C}{2}$
(d) $\cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$

19. A paticle moves towards east from a point A to a point B at the rate of 4 km/h and then towards north from B to C at the rate of 5km/hr. If AB = 12 km and BC = 5 km, then its average speed for its journey from A to C and resultant average velocity direct from A to C are respectively **[2004]**

(a) $\frac{13}{9}$ km/h and $\frac{17}{9}$ km/h
(b) $\frac{13}{4}$ km/h and $\frac{17}{4}$ km/h
(c) $\frac{17}{9}$ km/h and $\frac{13}{9}$ km/h
(d) $\frac{17}{4}$ km/h and $\frac{13}{4}$ km/h

20. A velocity $\frac{1}{4}$ m/s is resolved into two components along OA and OB making angles 30° and 45° respectively with the given velocity. Then the component along OB is **[2004]**

(a) $\frac{1}{8}(\sqrt{6}-\sqrt{2})$ m/s (b) $\frac{1}{4}(\sqrt{3}-1)$ m/s
(c) $\frac{1}{4}$ m/s (d) $\frac{1}{8}$ m/s

21. If t_1 and t_2 are the times of flight of two particles having the same initial velocity u and range R on the horizontal , then $t_1^2 + t_2^2$ is equal to **[2004]**

(a) 1 (b) $4u^2/g^2$
(c) $u^2/2g$ (d) u^2/g

22. Let R = {(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)} be a relation on the set $A = \{3, 6, 9, 12\}$. The relation is **[2005]**
(a) reflexive and transitive only
(b) reflexive only
(c) an equivalence relation
(d) reflexive and symmetric only

23. ABC is a triangle. Forces $\vec{P}$, $\vec{Q}$, $\vec{R}$ acting along IA, IB, and IC respectively are in equilibrium, where I is the incentre of ΔABC. Then $P:Q:R$ is **[2005]**
(a) $\sin A : \sin B : \sin C$
(b) $\sin\frac{A}{2} : \sin\frac{B}{2} : \sin\frac{C}{2}$
(c) $\cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$
(d) cos A : cos B : cos C

24. If in a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately **[2005]**
(a) 22.0 (b) 20.5 (c) 25.5 (d) 24.0

25. A lizard, at an initial distance of 21 cm behind an insect, moves from rest with an acceleration of $2\ cm/s^2$ and pursues the insect which is crawling uniformly along a straight line at a speed of 20 cm/s. Then the lizard will catch the insect after **[2005]**
(a) 20 s (b) 1 s (c) 21 s (d) 24 s

26. Two points A and B move from rest along a straight line with constant acceleration f and f' respectively. If A takes m sec. more than B and describes 'n' units more than B in acquiring the same speed then **[2005]**
(a) $(f - f')m^2 = ff'n$
(b) $(f + f')m^2 = ff'n$
(c) $\frac{1}{2}(f + f')m = ff'n^2$
(d) $(f' - f)n = \frac{1}{2}ff'm^2$

27. A and B are two like parallel forces. A couple of moment H lies in the plane of A and B and is contained with them. The resultant of A and B after combining is displaced through a distance **[2005]**
(a) $\frac{2H}{A-B}$
(b) $\frac{H}{A+B}$
(c) $\frac{H}{2(A+B)}$
(d) $\frac{H}{A-B}$

28. Let $x_1, x_2, \ldots\ldots\ldots\ldots x_n$ be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$. Then the possible value of n among the following is **[2005]**
(a) 15 (b) 18 (c) 9 (d) 12

29. A particle is projected from a point O with velocity u at an angle of 60° with the horizontal. When it is moving in a direction at right angles to its direction at O, its velocity then is given by **[2005]**
(a) $\frac{u}{3}$ (b) $\frac{u}{2}$ (c) $\frac{2u}{3}$ (d) $\frac{u}{\sqrt{3}}$

30. The resultant R of two forces acting on a particle is at right angles to one of them and its magnitude is one third of the other force. The ratio of larger force to the smaller one is **[2005]**
(a) $2:1$ (b) $3:\sqrt{2}$ (c) $3:2$ (d) $3:2\sqrt{2}$

31. ABC is a triangle, right angled at A. The resultant of the forces acting along $\overline{AB}, \overline{BC}$ with magnitudes $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively is the force along $\overline{AD}$, where D is the foot of the perpdicular from A onto BC. The magnitude of the resultant is **[2006]**
(a) $\frac{AB^2 + AC^2}{(AB)^2(AC)^2}$
(b) $\frac{(AB)(AC)}{AB + AC}$
(c) $\frac{1}{AB} + \frac{1}{AC}$
(d) $\frac{1}{AD}$

32. Let W denote the words in the English dictionary. Define the relation R by $R = \{(x, y) \in W \times W |$ the words x and y have at least one letter in common.} Then R is **[2006]**
(a) not reflexive, symmetric and transitive
(b) relexive, symmetric and not transitive
(c) reflexive, symmetric and transitive
(d) reflexive, not symmetric and transitive

33. Suppose a population A has 100 observations 101, 102, ..., 200 and another population B has 100 obsevrations 151, 152, ... 250. If V_A and V_B represent the variances of the two populations, respectively then $\frac{V_A}{V_B}$ is **[2006]**
(a) 1 (b) $\frac{9}{4}$ (c) $\frac{4}{9}$ (d) $\frac{2}{3}$

34. A particle has two velocities of equal magnitude inclined to each other at an angle θ. If one of them is halved, the angle between the other and the original resultant velocity is bisected by the new resultant. Then θ is **[2006]**
(a) 90° (b) 120° (c) 45° (d) 60°

35. A body falling from rest under gravity passes a certain point P. It was at a distance of 400 m from P, 4s prior to passing through P. If $g = 10 m/s^2$, then the height above the point P from where the body began to fall is **[2006]**
(a) 720m (b) 900m (c) 320m (d) 680m

36. The resultant of two forces Pn and 3n is a force of 7n. If the direction of 3n force were reversed, the resultant would be $\sqrt{19}\,n$. The value of P is **[2007]**

(a) $3\,n$ (b) $4\,n$ (c) $5\,n$ (d) $6\,n$.

37. A particle just clears a wall of height b at a distance a and strikes the ground at a distance c from the point of projection. The angle of projection is **[2007]**

(a) $\tan^{-1}\dfrac{bc}{a(c-a)}$ (b) $\tan^{-1}\dfrac{bc}{a}$

(c) $\tan^{-1}\dfrac{b}{ac}$ (d) 45°.

38. The average marks of boys in class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is **[2007]**

(a) 80 (b) 60 (c) 40 (d) 20.

39. A body weighing 13 kg is suspended by two strings 5m and 12m long, their other ends being fastened to the extremities of a rod 13m long. If the rod be so held that the body hangs immediately below the middle point, then tensions in the strings are **[2007]**

(a) 5 kg and 12 kg (b) 5 kg and 13 kg

(c) 12 kg and 13 kg (d) 5 kg and 5 kg

40. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b ? **[2008]**

(a) $a=0, b=7$ (b) $a=5, b=2$

(c) $a=1, b=6$ (d) $a=3, b=4$

41. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number if f y is a transcendental number". **[2008]**

Statement-1 : r is equivalent to either q or p

Statement-2 : r is equivalent to $\sim(p\leftrightarrow\sim q)$.

(a) Statement -1 is false, Statement-2 is true

(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1

(d) Statement -1 is true, Statement-2 is false

42. The statement $p\to(q\to p)$ is equivalent to **[2008]**

(a) $p\to(p\to q)$ (b) $p\to(p\vee q)$

(c) $p\to(p\wedge q)$ (d) $p\to(p\leftrightarrow q)$

43. **Statement-1 :** $\sim(p\leftrightarrow\sim q)$ is equivalent to $p\leftrightarrow q$.

Statement-2 : $\sim(p\leftrightarrow\sim q)$ is a tautology **[2009]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement -1

44. **Statement-1 :** The variance of first n even natural numbers is $\dfrac{n^2-1}{4}$.

Statement-2 : The sum of first n natural numbers is $\dfrac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\dfrac{n(n+1)(2n+1)}{6}$. **[2009]**

(a) Statement-1 is true, Statement-2 is true Statement-2 is not a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true. Statement-2 is a correct explanation for Statement-1.

45. If A, B and C are three sets such that $A\cap B = A\cap C$ and $A\cup B = A\cup C$, then **[2009]**

(a) $A=C$ (b) $B=C$

(c) $A\cap B=\phi$ (d) $A=B$

46. If the mean deviation of the numbers 1, 1 + d, 1 + 2d, ... 1 + 100d from their mean is 255, then d is equal to : **[2009]**

(a) 20.0 (b) 10.1 (c) 20.2 (d) 10.0

47. Let S be a non-empty subset of R. Consider the following statement :

P : There is a rational number $x\in$ S such that $x>0$.

Which of the following statements is the negation of the statement P ? **[2010]**

(a) There is no rational number $x\in$ S such than $x\le 0$.

(b) Every rational number $x\in$ S satisfies $x\le 0$.

(c) $x\in$ S and $x\le 0\Rightarrow x$ is not rational.

(d) There is a rational number $x\in$ S such that $x\le 0$.

48. Consider the following relations:

R = $\{(x, y)\,|\,x, y$ are real numbers and $x = wy$ for some rational number $w\}$;

$S=\left\{\left(\dfrac{m}{n},\dfrac{p}{q}\right)\,|\,m, n, p\right.$ and q are integers such that $n, q\neq 0$ and $qm=pn\}$.

Then **[2010]**

(a) Neither R nor S is an equivalence relation

(b) S is an equivalence relation but R is not an equivalence relation

(c) R and S both are equivalence relations

(d) R is an equivalence relation but S is not an equivalence relation

49. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is **[2010]**

(a) $\frac{11}{2}$ (b) 6 (c) $\frac{13}{2}$ (d) $\frac{5}{2}$

50. Let R be the set of real numbers.

Statement-1: $A = \{(x, y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R.

Statement-2: $B = \{(x, y) \in R \times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R. **[2011]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

51. Consider the following statements **[2011]**

P **:** Suman is brilliant
Q **:** Suman is rich
R **:** Suman is honest

The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as

(a) $\sim(Q \leftrightarrow (P \wedge \sim R))$ (b) $\sim Q \leftrightarrow \sim P \wedge R$
(c) $\sim(P \wedge \sim R) \leftrightarrow Q$ (d) $\sim P \wedge (Q \leftrightarrow \sim R)$

52. If the mean deviation about the median of the numbers a, $2a$,.......,$50a$ is 50, then $|a|$ equals **[2011]**

(a) 3 (b) 4 (c) 5 (d) 2

53. The negation of the statement **[2012]**
"If I become a teacher, then I will open a school", is :

(a) I will become a teacher and I will not open a school.
(b) Either I will not become a teacher or I will not open a school.
(c) Neither I will become a teacher nor I will open a school.
(d) I will not become a teacher or I will open a school.

54. Let $x_1, x_2,...., x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance. **[2012]**

Statement-1 : Variance of $2x_1, 2x_2, ..., 2x_n$ is $4\sigma^2$.

Statement-2 : Arithmetic mean $2x_1, 2x_2, ..., 2x_n$ is $4\bar{x}$.

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, statement-2 is true; statement-2 is ***not*** a correct explanation for Statement-1.
(d) Statement-1 is true, statement-2 is false.

55. Let $X = \{1,2,3,4,5\}$. The number of different ordered pairs (Y,Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty is : **[2012]**

(a) 5^2 (b) 3^5 (c) 2^5 (d) 5^3

56. Let A and B two sets containing 2 elements and 4 elements respectively. The number of subsets of A × B having 3 or more elements is **[JEE M 2013]**

(a) 256 (b) 220 (c) 219 (d) 211

57. Consider

Statement-1 : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement-2 : $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology. **[JEE M 2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-I.
(c) Statement-1 is true; Statement-2 is false.
(d) Statement-1 is false; Statement-2 is true.

58. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ? **[JEE M 2013]**

(a) mean (b) median
(c) mode (d) variance

59. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to: **[JEE M 2014]**

(a) X (b) Y (c) N (d) $Y - X$

60. The variance of first 50 even natural numbers is **[JEE M 2014]**

(a) 437 (b) $\frac{437}{4}$ (c) $\frac{833}{4}$ (d) 833

61. The statement $\sim(p \leftrightarrow \sim q)$ is : **[JEE M 2014]**

(a) a tautology (b) a fallacy
(c) eqivalent to $p \leftrightarrow q$ (d) equivalent to $\sim p \leftrightarrow q$

62. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set A × B, each having at least three elements is : **[JEE M 2015]**

(a) 275 (b) 510 (c) 219 (d) 256

63. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to : **[JEE M 2015]**

(a) $s \vee (r \vee \sim s)$ (b) $s \wedge r$
(c) $s \wedge \sim r$ (d) $s \wedge (r \wedge \sim s)$

64. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is: **[JEE M 2015]**

(a) 15.8 (b) 14.0 (c) 16.8 (d) 16.0

65. If $f(x)+2f\left(\frac{1}{x}\right)=3x, x\neq 0$ and

$S=\{x\in R: f(x)=f(-x)\}$; then S: **[JEE M 2016]**

(a) contains exactly two elements.
(b) contains more than two elements.
(c) is an empty set.
(d) contains exactly one element.

66. The Boolean Expression $(p\wedge\sim q)\vee q\vee(\sim p\wedge q)$ is equivalent to: **[JEE M 2016]**

(a) $p\vee q$ (b) $p\vee\sim q$ (c) $\sim p\wedge q$ (d) $p\wedge q$

67. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true? **[JEE M 2016]**

(a) $3a^2-34a+91=0$ (b) $3a^2-23a+44=0$
(c) $3a^2-26a+55=0$ (d) $3a^2-32a+84=0$

68. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is: **[JEE M 2016]**

(a) 20 (b) 5 (c) 6 (d) 10

22 Miscellaneous (Sets, Relations, Statistics & Mathematical Reasoning)

Section-A : JEE Advanced/ IIT-JEE

A	**1.**	n										
B	**1.**	F										
C	**1.**	(c)	**2.**	(b)	**3.**	(b)	**4.**	(c)	**5.**	(c)	**6.**	(d)
	7.	(d)	**8.**	(d)								
D	**1.**	(c)	**2.**	(b, c, d)								
E	**1.**	20	**3.**	$2(3\sqrt{13}+2\sqrt{2})\pi m^2$			**4.**	Correct mean 42.3 ; Correct variance 43. 81				
	7.	(i) 6 (ii) True (iii) Yes	**8.**	45			**9.**	$\sqrt{3}$				
	10.	new mean = 39.5, new variance = 49.25										
F	**1.**	A-s; B-t; C-r; D-r										
I	**1.**	4										

Section-B : JEE Main/ AIEEE

1.	(b)	**2.**	(a)	**3.**	(b)	**4.**	(a)	**5.**	(c)	**6.**	(c)
7.	(a)	**8.**	(a)	**9.**	(a)	**10.**	(c)	**11.**	(a)	**12.**	(b)
13.	(c)	**14.**	(c)	**15.**	(c)	**16.**	(c)	**17.**	(c)	**18.**	(d)
19.	(d)	**20.**	(a)	**21.**	(b)	**22.**	(a)	**23.**	(c)	**24.**	(d)
25.	(c)	**26.**	(d)	**27.**	(b)	**28.**	(b)	**29.**	(d)	**30.**	(d)
31.	(d)	**32.**	(b)	**33.**	(a)	**34.**	(b)	**35.**	(a)	**36.**	(c)
37.	(a)	**38.**	(a)	**39.**	(a)	**40.**	(d)	**41.**	(None)	**42.**	(d)
43.	(b)	**44.**	(c)	**45.**	(b)	**46.**	(b)	**47.**	(b)	**48.**	(b)
49.	(a)	**50.**	(b)	**51.**	(a)	**52.**	(b)	**53.**	(a)	**54.**	(d)
55.	(b)	**56.**	(c)	**57.**	(b)	**58.**	(d)	**59.**	(b)	**60.**	(d)
61.	(c)	**62.**	(c)	**63.**	(b)	**64.**	(b)	**65.**	(a)	**66.**	(a)
67.	(d)	**68.**	(b)								

A. Fill in the Blanks

1. Frequency for variable x is ${}^{n+x-1}C_x$ where $x = 0, 1, 2, ...n$.
Mode is the variable for which freq. is max.
Now, ${}^nC_r$ is max for $r = n/2$, if n is even

$r = \frac{n+1}{2}$ if n is odd.

If $n + x - 1$ is even then for max value of ${}^{n+x-1}C_x$,

$x = \frac{n+x-1}{2} \Rightarrow x = n-1$, $\therefore$ freq ${}^{2n-2}C_{n-1}$

If $n + x - 1$ is odd then for max value of ${}^{n+x-1}C_x$

$x = \frac{n+x-1+1}{2} \Rightarrow x = n$, $\therefore$ freq ${}^{2n-1}C_n$

But we know ${}^{2n-1}C_n = \frac{2n-1}{n}\,{}^{2n-2}C_{n-1}$

i.e., ${}^{2n-1}C_n > {}^{2n-2}C_{n-1}$

$\therefore$ Mode should be n.

B. True / False

1. Given that, $x * y = x - y + \sqrt{2}$

Consider $x = 2\sqrt{2}$, $y = \sqrt{2}$

then $x * y = 2\sqrt{2} - \sqrt{2} + \sqrt{2} = 3\sqrt{2}$ (irrational)

and $y * x = \sqrt{2} - 2\sqrt{2} + \sqrt{2} = 0$ (rational)

$\therefore x * y \neq y * x$

Hence $*$ is not symm. $\Rightarrow$ $*$ is not an equivalence relation

C. MCQs with ONE Correct Answer

1. **(c)** $X \cap (X \cup Y)^c = X \cap (X^c \cap Y^c) = (X \cap X^c) \cap Y^c$

$= \phi \cap Y^c = \phi$

2. **(b)** $\frac{12}{(3+\sqrt{5})+2\sqrt{2}} = \frac{12}{(3+\sqrt{5})+2\sqrt{2}} \times \frac{(3+\sqrt{5})-2\sqrt{2}}{(3+\sqrt{5})-2\sqrt{2}}$

$= \frac{12[3+\sqrt{5}-2\sqrt{2}]}{(3+\sqrt{5})^2-\left(2\sqrt{2}\right)^2} = \frac{12[3+\sqrt{5}-2\sqrt{2}]}{9+5+6\sqrt{5}-8}$

$= \frac{12[3+\sqrt{5}-2\sqrt{2}]}{6\left(\sqrt{5}+1\right)} \times \frac{\sqrt{5}-1}{\sqrt{5}-1}$

$= \frac{2[3\sqrt{5}+5-2\sqrt{10}-3-\sqrt{5}+2\sqrt{2}]}{5-1}$

$= \frac{2+2\sqrt{5}+2\sqrt{2}-2\sqrt{10}}{2} = 1+\sqrt{2}+\sqrt{5}-\sqrt{10}$

3. (b) If each of n observations is multiplied by a constant C, the standard deviation also gets multiplied by C.
4. (c) If s. d. = 0, statements like (a) and (b) can not be given.
5. (c) Given that $x_1 < x_2 < x_3 < .... < x_{201}$

$\therefore$ Median of the given observation $= \dfrac{201+1}{2}$ th items

$= 101$th item $= x_{101}$

Now, deviations will be minimum if taken from the median

$\therefore$ Mean deviation will be min if $k = x_{101}$.

6. (d) If any of the inequations hold, it must hold for any real numbers $x_1, x_2, ... x_n$ and any $n \in N$.

$\therefore$ let $x_1 = 1, x_2 = 2, x_3 = 3; n = 3$ then we can check none of the inequalities (a), (b) or (c) are satisfied.

7. (d) $S = \{1, 2, 3, 4\}$

Let P and Q be disjoint subsets of S

Now for any element $a \in s$, following cases are possible

$a \in P$ and $a \notin Q$, $a \notin P$ and $a \in Q$, $a \notin P$ and $a \notin Q$

$\Rightarrow$ For every element there are three option

$\therefore$ Total options $= 3^4 = 81$

Here $P \neq Q$ except when $P = Q = \phi$

$\therefore$ 80 ordered pairs (P, Q) are there for which $P \neq Q$.

Hence total number of unordered pairs of disjoint subsets $= \dfrac{80}{2} + 1 = 41$

8. (d) $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$

$\sin\theta = (\sqrt{2}+1)\cos\theta, \tan\theta = \sqrt{2}+1$

$Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$

$\cos\theta = (\sqrt{2}-1)\sin\theta$ or $\tan\theta = \sqrt{2}+1$

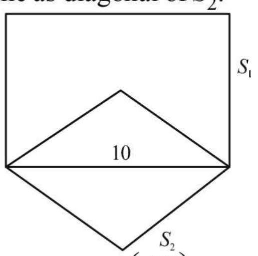
$\therefore P = Q$

D. MCQs with ONE or MORE THAN ONE Correct

1. (c) Let n be the number of newspapers which are read. Then $60\, n = (300)(5) \Rightarrow n = 25$
2. (b, c, d)

Given side of square $S_1 = 10$ cm

As it is same as diagonal of S_2.



$\therefore$ Side of square $S_2 = \left(\dfrac{10}{\sqrt{2}}\right)$ cm

Similarly, side of square $S_3 = \dfrac{10}{(\sqrt{2})^2}$ cm

In the same manner, side of square $S_n = \dfrac{10}{(\sqrt{2})^{n-1}}$ cm

$\therefore$ Area of square of side $S_n = \left[\dfrac{10}{(\sqrt{2})^{n-1}}\right]^2 = \left[\dfrac{10}{(\sqrt{2})^{n-1}}\right] < 1$

$\Rightarrow 100 < (2)^{n-1} \Rightarrow 2^{n-1} > 100 \Rightarrow n - 1 \geq 7 \Rightarrow n \geq 8$

E. Subjective Problems

1. We have

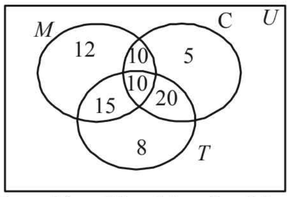
$n(U) = 100$, where U stands for universal set

$n(M \cap C \cap T) = 10$; $n(M \cap C) = 20$;.

$n(C \cap T) = 30$; $n(M \cap T) = 25$;

n (M only) $= 12$; n (only C) $= 5$; n (only T) $= 8$

Filling all the entries we obtain the Venn diagram as shown :



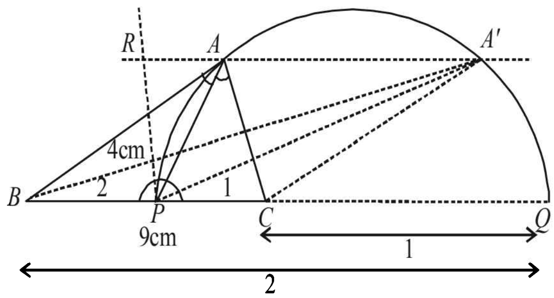
$\therefore\ n(M \cap C \cup T) = 12 + 10 + 5 + 15 + 10 + 20 + 8 = 80$

$\therefore\ n(M \cup C \cup T)' = 100 - 80 = 20$

2. (a) To construct a Δ with base = 9 cm, altitude = 4 cm and ratio of the other two sides as 2 : 1.

Steps of Construction :

1. Draw $BC = 9$ cm
2. Divide BC internally at P and externally at Q in the ratio 2 : 1.



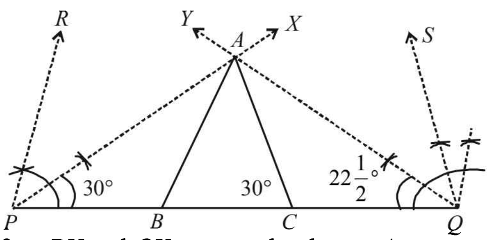
3. Draw a semicircle on PQ.
4. Draw a line $\parallel BQ$ at a distance of 4 cm from intersecting semicircle at A and A'.
5. ABC and $A'BC$ are the required Δ's.

(b) To construct a Δ with perimeter = 15 cm, base angles 60° and 45°.

Steps of Construction :

1. Draw $PQ = 15$ cm
2. At P draw $\angle RPQ = 60°$ and $\angle XPQ = \dfrac{60°}{2} = 30°$

and at Q draw $\angle SQP = 45°$ and $\angle YQP = \dfrac{45°}{2} = 22\dfrac{1}{2}°$



3. PX and QY meet each other at A.
4. Through A draw $AB \parallel PR$ and $AC \parallel QS$.
5. ABC is the required Δ.

Justification: $\because AB \parallel PR$ and PA transversal

$\therefore\ \angle PAB = \angle RPA = \dfrac{1}{2} \times 60° = 30°$

$\angle APB = \angle BAP = 30° \Rightarrow AB = PB$

Similarly $AC = CQ$

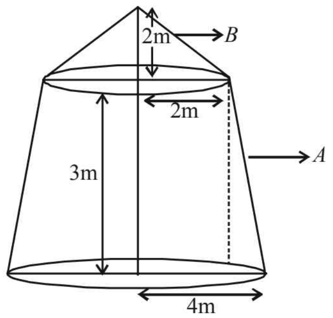
$\therefore \quad AB + BC + CA = PB + BC + CQ = 15$ cm

Also $\angle ABC = \angle RPB = 60°$ and $\angle ACB = \angle SQS = 45°$

3. Slant height of frustum A $= \sqrt{(4-2)^2 + 3^2} = \sqrt{13}$

$\therefore$ Curved surface area of frustum $= \pi(4+2)\sqrt{13}$

$= 6\sqrt{13}\pi\ m^2$



Also slant height of cone $B = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

$\therefore$ Curved surface area of cone $= \pi \times 2 \times 2\sqrt{2} = 4\sqrt{2}\pi\ m^2$

$\therefore$ Area of canvas required $= 6\sqrt{13}\pi + 4\sqrt{2}\pi$

$= 2\left(3\sqrt{13} + 2\sqrt{2}\right)\pi\ m^2.$

4. Let the remaining 9 readings be $x_1, x_2, x_3 \ldots\ldots x_9$ and tenth is taken by the student as 52.

$\therefore$ Incorrect mean $= \dfrac{x_1 + x_2 + \ldots + x_9 + 52}{10} = 45$

$\Rightarrow x_1 + x_2 + \ldots + x_9 = 450 - 52 = 398 \qquad \ldots(1)$

$\therefore$ Correct mean $= \dfrac{x_1 + x_2 + \ldots + x_9 + 25}{10} = \dfrac{398 + 25}{10}$

$\Rightarrow$ Correct mean $= 42.3$

Incorrect variance $= \dfrac{\Sigma(x_i - \bar{x})^2}{n}$

$\Rightarrow 16 = \dfrac{(x_1-45)^2 + (x_2-45)^2 + \ldots + (x_9-45)^2 + (52-45)^2}{10}$

$\Rightarrow (x_1-45)^2 + (x_2-45)^2 + \ldots + (x_9-45)^2 = 160 - 49 = 111$

$\Rightarrow (x_1^2 + x_2^2 + \ldots + x_9^2) - 90\,(x_1 + x_2 \ldots + x_9) + 9 \times (45)^2 = 111$

$\Rightarrow x_1^2 + x_2^2 + \ldots + x_9^2$

$= 111 + 90 \times 398 - 9 \times (45)^2 = 17706 \qquad \ldots(2)$

Correct variance $= \dfrac{\Sigma(x_i - 42.3)^2}{10}$

$= \dfrac{(x_1 - 42.3)^2 + (x_2 - 42.3)^2 + \ldots + (x_9 - 42.3)^2 + (25 - 42.3)^2}{10}$

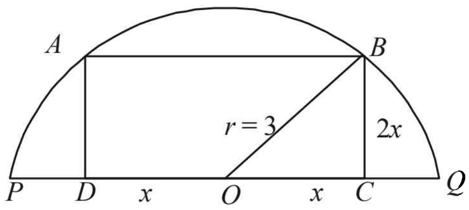
$= \dfrac{(x_1^2 + x_2^2 + \ldots + x_9^2) - 84.6(x_1 + x_2 + \ldots + x_9) + 9 \times (42.3)^2 + 299.29}{10}$

$= \dfrac{17706 - 84.6 \times 398 + 16103.61 + 299.29}{10}$

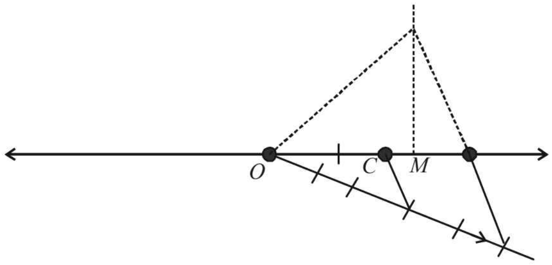
$= \dfrac{34108.9 - 33670.8}{10} = \dfrac{438.1}{10} = 43.81$

5. The rough figure is as shown, let $2x$ be the side of square.

In ΔOBC $r^2 = x^2 + 4x^2$



$\Rightarrow x^2 = \dfrac{r^2}{5} \Rightarrow x = \dfrac{r}{\sqrt{5}} = \dfrac{3}{\sqrt{5}} = \dfrac{3\sqrt{5}}{5}$

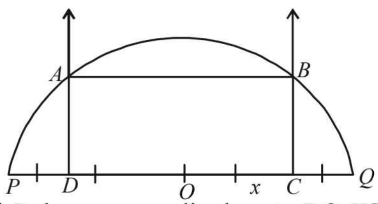


Now on number line we locate M such that $OM = \sqrt{5}$

Divide OM in five equal parts and take a point C on it such that $OC : CM = 3 : 2$. So, that $OC = \dfrac{3}{5}\sqrt{5}$

Now, draw a line segment $PQ = 6$ cm whose mid-point is O.

From this cut a line segment OC $= \dfrac{3\sqrt{5}}{5}$ and $OD = \dfrac{3\sqrt{5}}{5}$ on opposite sides of O.



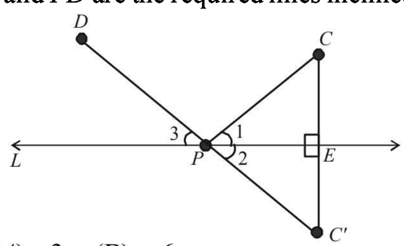
At C and B draw perpendiculars to PQ. With O as centre and OP as radius, draw a semicircle intersecting the perpendiculars draw at C & D at A and P respectively. Join AB. $ABCD$ is the required square.

6. Here we are given two points C and D on the same side of line L. To find a point P on L such that PC and PD are equally inclined to L.

Steps of Construction :

1. From C draw $CE \perp L$ and produce it to C' such that $EC' = EC$.
2. Join $C'D$ intersecting L at P. Also join CP.
3. By simple geometry $\angle 1 = \angle 2$ and $\angle 2 = \angle 3 \Rightarrow \angle 1 = \angle 3$

$\therefore$ PC and PD are the required lines inclined equally to L.



7. (i) $n(A) = 3$, n (B) $= 6$

We know that $n(A \cup B) \geq \max(n(A), n(B))$

$\Rightarrow n(A \cup B) \geq 6$

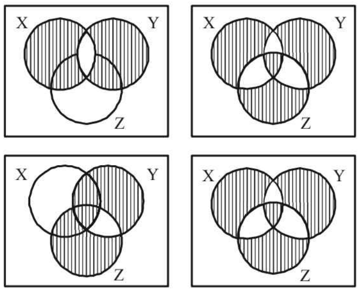
$\therefore$ Min number of element that $A \cup B$ can have is 6.

(ii) Here $R \times (P^c \cup Q^c)^c = R \times (P \cap Q)$

$= (R \times P) \cap (R \times Q)$

$\therefore$ Given equality is true.

(iii) Yes



From (1) and (2) $(X \circ Y) \circ Z = X \circ (Y \circ Z)$

8. We are given that $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ ....(1)

Each $A_i'^{s}$ contain 5 elements, so $\bigcup_{i=1}^{30} A_i$ contains $5 \times 30 = 150$ elements (with repetition) out of which each element is repeated 10 times, (as given that each element of S belongs to 10 $A_i'^{s}$)

$\therefore$ Number of different elements in $\bigcup_{i=1}^{30} A_i$ is $= \frac{150}{10} = 15$

$\therefore$ From eqn (1) we can say S contains 15 elements...(2)

Again each $B_j'^{s}$ contains 3 elements, so $\bigcup_{j=1}^{n} B_j$ contains $3 \times n = 3n$ elements (with repetition), out of which each element is repeated 9 times (as each element of S belongs to 9 $B_j'^{s}$)

$\therefore$ No. of different elements in $\bigcup_{j=1}^{n} B_j = \frac{3n}{9} = \frac{n}{3}$

$\therefore$ From eqn (1) we can say S contain $n/3$ elements....(3)

From (2) and (3) we get $\frac{n}{3} = 15 \Rightarrow n = 45$

9. Given that : Mean square deviation for the observations $x_1, x_2, \ldots, x_n$, about a point. c is given by $\frac{1}{n}\sum_{i=1}^{n}(x_i - c)^2$.

Also given that mean square deviations about – 1 and + 1 are 7 and 3 for a particular set of observations.

$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}(x_i+1)^2 = 7$ and $\frac{1}{n}\sum_{i=1}^{n}(x_i-1)^2 = 3$

$\Rightarrow \sum_{i=1}^{n}(x_i^2+2x_i+1) = 7n$ and $\sum_{i=1}^{n}(x_i^2-2x_i+1) = 3n$

$\Rightarrow \sum x_i^2 + 2\sum x_i + n = 7n$ and **NOTE THIS STEP**

$\sum x_i^2 - 2\sum x_i + n = 3n$

$\Rightarrow \sum x_i^2 + 2\sum x_i = 6n$ ...(1)

and $\sum x_i^2 - 2\sum x_i = 2n$ ...(2)

Subtracting (2) from (1), we get

$4\sum x_i = 4n \Rightarrow \frac{\sum x_i}{n} = 1 \Rightarrow \bar{x} = 1$

Now standard deviation for same set of observations

$= \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-1)^2} = \sqrt{3}$

{using the given value}

10. $n = 40$, $\bar{x} = 40$, Var. $= 49$

$\frac{\sum f_i x_i}{40} = \bar{x} = 40 \Rightarrow \sum f_i x_i = 1600$ ....(1)

Also Var. $= 49 = \frac{1}{40}\sum f_i(x_i-40)^2$

$\Rightarrow 49 = \frac{1}{40}\sum f_i x_i^2 - 2\sum f_i x_i + 40\sum f_i$

$\Rightarrow 49 = \frac{1}{40}\sum f_i x_i^2 - 2 \times 1600 + 40 \times 40$

$\Rightarrow \frac{1}{40}\sum f_i x_i^2 = 1649$ ...(2)

Let 21 – 30 and 31 – 40 denote the k^{th} and $(k + 1)^{th}$ class intervals respectively.

Then if before correction f_k and f_{k+1} are the frequencies of these intervals then after correction (2 observations are shifted from 31 – 40 to 21 – 30), frequency of k^{th} intervals becomes $f_k + 2$ and frequency of $(k + 1)^{th}$ interval becomes $f_{k+1} - 2$.

Then, we get

$\bar{x}_{new} = \frac{1}{40}\sum_{i=1}^{40} f_i x_i + \frac{2}{40}(x_k - x_{k+1})$

$= \frac{1}{40}\sum_{i=1}^{40} f_i x_i + \frac{1}{20}(-10) = 40 - 0.5 = 39.5$

$Var_{new} = \frac{1}{40}\left[\sum_{\substack{i=1\\ i\neq k,k+1}}^{40} f_i(x_i-39.5)^2 + f_k(x_k-39.5)^2 + f_{k+1}(x_{k+1}-39.5)^2\right]$

$= \frac{1}{40}\left[\sum_{\substack{i=1\\ i\neq k,k+1}}^{40} (f_i x_i^2 - 79 f_i x_i + 39.5)^2 f_i)\right]$

$+\frac{1}{40}\left[f_k x_k^2 - 79 f_k x_k + (39.5)^2 f_k + f_{k+1}x_k^2 + 1\right.$
$\left. - 79 f_{k+1} x_{k+1} + (39.5)^2 f_{k+1}\right]$

$= \frac{1}{40}\sum_{i=1}^{40} f_i x_i^2 - 79.\frac{1}{40}\sum_{i=1}^{40} f_i x_i + (39.5)^2 \frac{1}{40}\sum f_i$

$= 1649 - 3160 + 1560.25 = 49.25$ [Using eqn. (1) and (2)]

11. Given that $z_1 R z_2$ iff $\frac{z_1 - z_2}{z_1 + z_2}$ is real.

To show that R is an equivalence relation.

Reflexivity : For $z_1 = z_2 = z$ (say)

$\frac{z_1 - z_2}{z_1 + z_2} = \frac{z - z}{z + z} = 0$ which is real

$\therefore$ $z R z \ \forall \ z$ $\therefore$ R is reflexive.

Symmetric : Let $z_1 R z_2 \Rightarrow \frac{z_1 - z_2}{z_1 + z_2}$ is real $\Rightarrow -\left(\frac{z_1 - z_2}{z_1 + z_2}\right)$ is also real

$\Rightarrow \frac{z_2 - z_1}{z_2 + z_1}$ is real $\Rightarrow z_2 R z_1 \Rightarrow$ R is symmetric.

Transitivity : Let $z_1 R z_2$ and $z_2 R z_3$

$\Rightarrow \frac{z_1 - z_2}{z_1 + z_2}$ is real and $\frac{z_2 - z_3}{z_2 + z_3}$ is also real

Now, $\frac{z_1 - z_2}{z_1 + z_2}$ is real $\Rightarrow I_m\left(\frac{z_1 - z_2}{z_1 + z_2}\right) = 0$

$\Rightarrow I_m\left(\frac{(x_1 - x_2) + i(y_1 - y_2)}{(x_1 + x_2) + i(y_1 + y_2)}\right) = 0$

$\Rightarrow I^m((x_1 - x_2) + i(y_1 - y_2))((x_1 + x_2) - i(y_1 + y_2)) = 0$

$\Rightarrow (x_1 + x_2)(y_1 - y_2) - (x_1 - x_2)(y_1 + y_2) = 0$

$\Rightarrow x_2 y_1 - x_1 y_2 = 0$

$\Rightarrow \frac{x_1}{y_1} = \frac{x_2}{y_2}$ ...(1)

Similarly, $I_m\left(\frac{z_2 - z_3}{z_2 + z_3}\right) = 0 \Rightarrow \frac{x_2}{y_2} = \frac{x_3}{y_3}$ ...(2)

From (1) and (2) we get $\frac{x_1}{y_1} = \frac{x_3}{y_3}$

$\Rightarrow I_m\left(\frac{z_1 - z_3}{z_1 + z_3}\right) = 0 \Rightarrow \frac{z_1 - z_3}{z_1 + z_3}$ is real

$\Rightarrow z_1 R z_3 \therefore R$ is transitive.

Thus R is reflexive, symmetric and transitive.

Hence R is an equivalence relation.

F. Match the Following

1. **A → (s), B → (t), C → (r) D → (r)**

Let $z = x + iy$ where and $x^2 + y^2 = 1$ and $x \neq \pm 1$

Then $\text{Re}\left(\frac{2iz}{1-z^2}\right) = \text{Re}\left(\frac{2i(x+iy)}{1-(x^2-y^2+2ixy)}\right)$

$= \text{Re}\left(\frac{-2y+2ix}{1-x^2+y^2-2ixy}\right) = \text{Re}\left(\frac{-2y+2ix}{2y(y-ix)}\right)$

$= \text{Re}\left(\frac{-1}{y}\right) = \frac{-1}{y}$

where $-1 \leq y \leq | \Rightarrow \frac{-1}{y} \geq 1$ of $\frac{-1}{y} \leq -1$

$\therefore \text{Re}\left(\frac{2iz}{1-z^2}\right) \in (-\infty, -1] \cup [1, \infty)$ $\therefore$ **A → s**

(B) For the domain of $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$

We should have

$-1 \leq \left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right) \leq 1 \Rightarrow -1 \leq \frac{8.3^x}{9-3^{2x}} \leq 1$

Let $3^x = t$ then $-1 \leq \frac{8t}{9-t^2} \leq 1$

$\Rightarrow \frac{8t}{9-t^2} \geq -1$ and $\frac{8t}{9-t^2} \leq 1$

$\Rightarrow \frac{8t+9-t^2}{9-t^2} \geq 0$ and $\frac{8t-9+t^2}{9-t^2} \leq 0$

$\Rightarrow \frac{t^2-8t-9}{t^2-9} \geq 0$ and $\frac{t^2+8t-9}{t^2-9} \geq 0$

$\Rightarrow \frac{(t-9)(t+1)}{(t-3)(t+3)} \geq 0$ and $\frac{(t+9)(t-1)}{(t+3)(t-3)} \geq 0$

Also $t = 3^x$ can not be – ve

$\therefore t \in (0,3) \cup [9,\infty)$ and $t \in (0,1] \cup [3,\infty)$

$\Rightarrow x \in (-\infty,1) \cup [2,\infty)$ and $x \in (-\infty,0] \cup (1,\infty)$

Combining the two, we get $x \in (-\infty,0] \cup [2,\infty)$

$\therefore$ **B → t**

(C) $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$

$R_1 + R_3$

$= \begin{vmatrix} 0 & 0 & 2 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$

$= 2(1+\tan^2\theta) = 2\sec^2\theta \in [2,\infty)$ for $0 \leq \theta < \frac{\pi}{2}$

$\therefore$ **C → r**

(D) $f(x) = x^{3/2}(3x-10), x \geq 0$

$\therefore f'(x) = \frac{3}{2}\sqrt{x}(3x-10) + 3x\sqrt{x}$

For f(x) to be increasing $f'(x) \geq 0$

$\Rightarrow 3\sqrt{x}[3x-10+2x] \geq 0$

$\Rightarrow \sqrt{x}(5x-10) \geq 0$ but $x \geq 0 \Rightarrow x \geq 2$

$\therefore f(x)$ is incresing on $[2,\infty)$

$\therefore$ **D → r.**

I. Integer Value Correct Type

1. **(4)** Let $\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots\ldots}}} = y$

$\Rightarrow 4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots\ldots}} = y^2$

$\Rightarrow 4-\frac{y}{3\sqrt{2}} = y^2 \Rightarrow y^2 + \frac{y}{3\sqrt{2}} - 4 = 0$

$\Rightarrow 3\sqrt{2}y^2 + y - 12\sqrt{2} = 0 \Rightarrow y = \frac{-1+\sqrt{1+288}}{6\sqrt{2}}$

(rejecting -ve value as y is a +ve square)

$\Rightarrow y = \frac{-1+17}{6\sqrt{2}} = \frac{8}{3\sqrt{2}}$

$\therefore$ The required value is

$6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}} \times \frac{8}{3\sqrt{2}}\right) = 6 + \log_{3/2}\frac{4}{9}$

$= 6 + \frac{2\log_{2/3}}{\log_{3/2}} = 6 - 2 = 4$

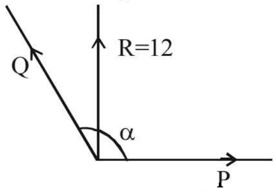
Section-B JEE Main/ AIEEE

1. **(b)** Total student = 100;
for 70 stds. total marks = 75 × 70 = 5250
$\Rightarrow$ Total marks of girls = 7200 – 5250 = 1950

Average of girls $= \frac{1950}{30} = 65$

2. **(a)** Given $P + Q = 18$ ....(1)
$P^2 + Q^2 + 2PQ\cos\alpha = 144$ ......(2)

$\tan 90° = \frac{Q\sin\alpha}{P + Q\cos\alpha}$

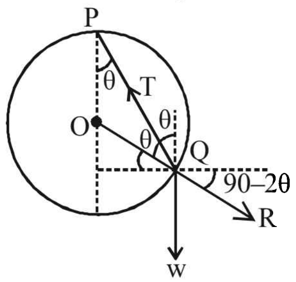


$\Rightarrow$ $P + Q\cos\alpha = 0$ ......(3)
From (2) and (3),
$Q^2 - P^2 = 144 \Rightarrow (Q-P)(Q+P) = 144$

$\therefore$ $Q - P = \frac{144}{18} = 8$

From (1), On solving, we get $Q = 13, P = 5$

3. **(b)** $\angle TQW = 180 - \theta$; $\angle RQW = 2\theta$; $\angle RQT = 180 - \theta$



Applying Lami's theorem at Q,

$$\frac{T}{\sin 2\theta} = \frac{R}{\sin(180-\theta)} = \frac{W}{\sin(180-\theta)}$$

$\Rightarrow$ $R = W$ and $T = 2W\cos\theta$

4. **(a)** $n = 9$ then median term $= \left(\frac{9+1}{2}\right)^{th} = 5^{th}$ term. Last four observations are increased by 2. The median is 5th observation which is remaining unchanged.
$\therefore$ there will be no change in median.

5. **(c)** $\vec{G} = \vec{r} \times \vec{p}$; $|\vec{G}| = rp\sin\theta$

$|\vec{H}| = rp\cos\theta$ $[\because \sin(90^o + \theta) = \cos\theta]$

$G = rp\sin\theta$.......(1) $H = rp\cos\theta$............(2)

$x = rp\sin(\theta + \alpha)$......(3)

From (1), (2) & (3), $x = \vec{G}\cos\alpha + \vec{H}\sin\alpha$.

6. **(c)** $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ ....(1)

$4R^2 = P^2 + 4Q^2 + 4PQ\cos\theta$ ....(2)

$4R^2 = P^2 + Q^2 - 2PQ\cos\theta$ ....(3)

On (1) + (3), $5R^2 = 2P^2 + 2Q^2$ ....(4)

On (3) × 2 + (2), $12R^2 = 3P^2 + 6Q^2$ .....(5)

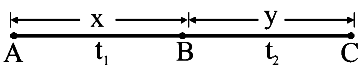
$2P^2 + 2Q^2 - 5R^2 = 0$ .....(6)

$3P^2 + 6Q^2 - 12R^2 = 0$ .....(7)

$$\frac{P^2}{-24+30} = \frac{Q^2}{24-15} = \frac{R^2}{12-6}$$

$$\frac{P^2}{6} = \frac{Q^2}{9} = \frac{R^2}{6} \text{ or } P^2 : Q^2 : R^2 = 2:3:2$$

7. **(a)** Let the body travels from A to B with constant acceleration t and from B to C with constant retardation r.



If $AB = x$, $BC = y$, time taken from A to $B = t_1$ and time taken from B to $C = t_2$, then $s = x + y$ and $t = t_1 + t_2$
For the motion from A to B

$v^2 = u^2 + 2fs \Rightarrow v^2 = 2fx \ (\because u = 0)$

$\Rightarrow$ $x = \frac{v^2}{2f}$ ....(1)

and $v = u + ft$ $\Rightarrow v = ft_1$

$\Rightarrow$ $t_1 = \frac{v}{f}$ ...(2)

For the motion from B to C

$t_2 = \frac{v}{r}$

Adding equations (1) and (3), we get

$$x + y = \frac{v^2}{2}\left[\frac{1}{f} + \frac{1}{r}\right] = s$$

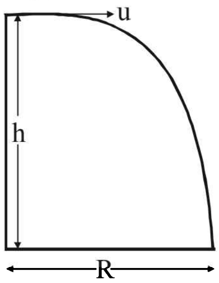
Adding equations (2) and (4), we get

$$t_1 + t_2 = v\left[\frac{1}{f} + \frac{1}{r}\right] = t$$

$$\therefore \quad \frac{t^2}{2s} = \frac{v^2\left[\frac{1}{f} + \frac{1}{r}\right]^2}{2 \times \frac{v^2}{2}\left(\frac{1}{f} + \frac{1}{r}\right)} = \frac{1}{f} + \frac{1}{r}$$

$$\Rightarrow \quad t = \sqrt{2s\left(\frac{1}{f} + \frac{1}{r}\right)}$$

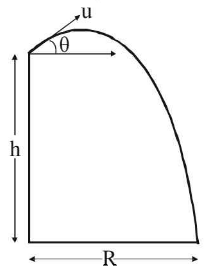
8. **(a)** For the stone projected horizontally, for horizontal motion, using distance
= speed × time $\Rightarrow R = ut$
and for vertical motion



$h = 0 \times t + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

$\therefore$ We get $R = u\sqrt{\frac{2h}{g}}$ ....(1)

For the stone projected at an angle θ, for horizontal and vertical motions, we have



$R = u\cos\theta \times t$ ....(2)

and $h = -u\sin\theta \times t + \frac{1}{2}gt^2$ ....(3)

From (1) and (2) we get

$\Rightarrow u\sqrt{\frac{2h}{g}} = u\cos\theta \times t \Rightarrow t = \frac{1}{\cos\theta}\sqrt{\frac{2h}{g}}$

Substituting this value of t in eq (3) we get

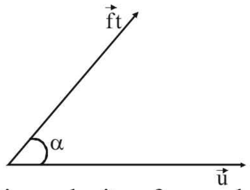
$h = -\frac{u\sin\theta}{\cos\theta}\sqrt{\frac{2h}{g}} + \frac{1}{2}g\left[\frac{2h}{g\cos^2\theta}\right]$

$h = -u\sqrt{\frac{2h}{g}}\tan\theta + h\sec^2\theta$

$h = -u\sqrt{\frac{2h}{g}}\tan\theta + h\tan^2\theta + h$

$\tan^2\theta - u\sqrt{\frac{2}{hg}}\tan\theta = 0; \therefore \tan\theta = u\sqrt{\frac{2}{hg}}$

9. (a) We can consider the two velocities as $\vec{v}_1 = u\hat{i}$ and $\vec{v}_2 = (ft\cos\alpha)\hat{i} + (ft\sin\alpha)\hat{j}$



$\therefore$ Relative velocity of second with respect to first

$\vec{v} = \vec{v}_2 - \vec{v}_1 = (ft\cos\alpha - u)\hat{i} + ft\sin\alpha\,\hat{j}$

$\Rightarrow |\vec{v}|^2 = (ft\cos\alpha - u)^2 + (ft\sin\alpha)^2$

$= f^2t^2 + u^2 - 2uft\cos\alpha$

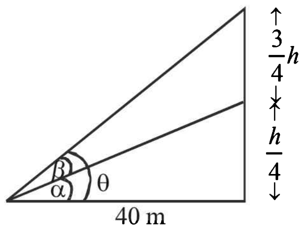
For $|\vec{v}|$ to be min we should have

$\frac{d|v|^2}{dt} = 0 \Rightarrow 2f^2t - 2uf\cos\alpha = 0 \Rightarrow t = \frac{u\cos\alpha}{f}$

Also $\frac{d^2|v|^2}{dt^2} = 2f^2 = +ve$

$\therefore |v|^2$ and hence $|v|$ is least at the time $\frac{u\cos\alpha}{f}$

10. (c)



$\theta = \alpha + \beta, \beta = \tan^{-1}\left(\frac{3}{5}\right)$ or $\beta = \theta - \alpha$

$\Rightarrow \tan\beta = \frac{\tan\theta - \tan\alpha}{1 + \tan\theta.\tan\alpha}$ or $\frac{3}{5} = \frac{\frac{h}{40} - \frac{h}{160}}{1 + \frac{h}{40}.\frac{h}{160}}$

$h^2 - 200h + 6400 = 0 \Rightarrow h = 40$ or 160 metre

$\therefore$ possible height $= 40$ metre

11. (a) Let β be the inclination of the plane to the horizontal and u be the velocity of projection of the projectile

$R_1 = \frac{u^2}{g(1+\sin\beta)}$ *and* $R_2 = \frac{u^2}{g(1-\sin\beta)}$

$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2g}{u^2}$ *or* $\frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{R}\left[\because R = \frac{u^2}{g}\right]$

$\therefore R_1, R, R_2$ *are in H.P.*

12. (b) $\Sigma x = 170, \Sigma x^2 = 2830$ increase in $\Sigma x = 10$, then

$\Sigma x' = 170 + 10 = 180$

Increase in $\Sigma x^2 = 900 - 400 = 500$ then

$\Sigma x'^2 = 2830 + 500 = 3330$

Variance $= \frac{1}{n}\Sigma x'^2 - \left(\frac{1}{n}\Sigma x'\right)^2 = \frac{1}{15} \times 3330 - \left(\frac{1}{15} \times 180\right)^2$

$= 222 - 144 = 78$

13. (c) $\because (1,1) \notin R \Rightarrow R$ is not reflexive $(2,3) \in R$ but $(3,2) \notin R$

$\therefore$ R is not symmetric

14. (c) Only first (A) and second (B) statements are correct.

15. (c) Clearly mean $A = 0$

Standard deviation $\sigma = \sqrt{\frac{\sum(x-A)^2}{2n}}$

$2 = \sqrt{\frac{(a-0)^2 + (a-0)^2 + ...(0-a)^2 + ...}{2n}} = \sqrt{\frac{a^2.2n}{2n}} = |a|$

Hence $|a| = 2$

16. (c) Let forces be P and Q. then $P + Q = 4$ ....(1)

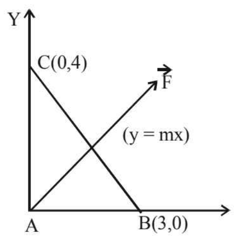
and $P^2 + Q^2 = 3^2$ ....(2)

Solving we get the forces

$\left(2 + \frac{\sqrt{2}}{2}\right)N$ and $\left(2 - \frac{\sqrt{2}}{2}\right)N$

17. (c) Since, the moment about A is zero, hence $\vec{F}$ passes through A. Taking A as origin. Let the line of action of force $\vec{F}$ be $y = mx$.(see *figure*)

Moment about $B = \frac{3m}{\sqrt{1+m^2}}|\vec{F}| = 9$ ....(1)



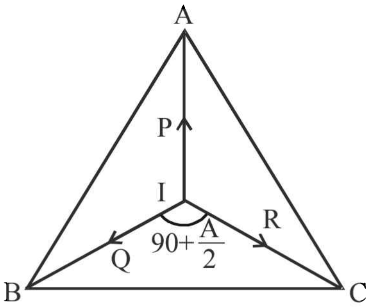
Moment about $C = \frac{4}{\sqrt{1+m^2}}|\vec{F}| = 16$ ....(2)

Dividing (1) by (2), we get $m = \frac{3}{4} \Rightarrow |\vec{F}| = 5N.$

18. (d) IA, IB, IC are bisectors of the angles A, B and C as I is incentre of ΔABC.

Now $\angle BIC = 180 - \frac{B}{2} - \frac{C}{2} = 90° + \frac{A}{2}$ etc.

Applying Lami's theorem at I

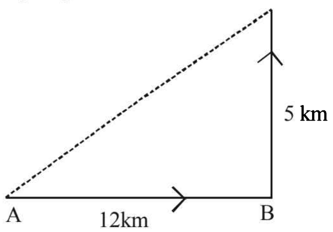


$$\frac{P}{Sin(90° + \frac{A}{2})} = \frac{Q}{\sin\left(90° + \frac{B}{2}\right)} = \frac{R}{\sin\left(90° + \frac{C}{2}\right)}$$

$\Rightarrow \quad P : Q : R = \cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$

19. (d) Time taken by the particle in complete journey

$T = \frac{12}{4} + \frac{5}{5} = 4\ hr.$



$\therefore$ Average speed $= \frac{12+5}{4} = \frac{17}{4}$

Average velocity $= \sqrt{\frac{12^2 + 5^2}{4}} = \frac{13}{4}$

[using vector addition]

20. (a) If $v = \frac{1}{4}$, component along OB

$$= \frac{v\sin 30°}{\sin(45° + 30°)} = \frac{\frac{1}{4} \times \frac{1}{2}}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{\sqrt{6} - \sqrt{2}}{8}$$

21. (b) For same horizontal range the angles of projection must be α and $\frac{\pi}{2} - \alpha$

$$\therefore t_1 = \frac{2u\sin\alpha}{g} \text{ and } t_2 = \frac{2u\sin\left(\frac{\pi}{2} - \alpha\right)}{g} = \frac{2u\cos\alpha}{g}$$

$$\therefore t_1^2 + t_2^2 = \frac{4u^2}{g^2}$$

22. (a) Reflexive and transitive only.

e.g. (3, 3), (6, 6), (9, 9), (12, 12) [Reflexive]

(3, 6), (6, 12), (3, 12) [Transitive].

$(3, 6) \in R$ but $(6, 3) \notin R$ [non symmetric]

23. (c) Similar to Question 18

24. (d) Mode + 2Mean = 3 Median

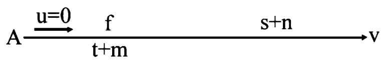
$\Rightarrow$ Mode $= 3 \times 22 - 2 \times 21 = 66 - 42 = 24.$

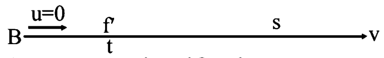
25. (c) Let the lizard catches the insect after time t then distance covered by lizard = 21cm + distance covered by insect

$$\Rightarrow \frac{1}{2}ft^2 = 4 \times t + 21 \Rightarrow \frac{1}{2} \times 2 \times t^2 = 20 \times t + 21$$

$$\Rightarrow t^2 - 20t - 21 = 0 \Rightarrow t = 21 \text{ sec}$$

26. (d)





As per question if point B moves s distance in t time then point A moves $(s + n)$ distance in time $(t + m)$ after which both have same velocity v.

Then using equation $v = u + at$ we get

$$v = f(t + m) = f't \Rightarrow t = \frac{f\ m}{f' - f} \qquad ....(1)$$

Using equation $v^2 = u^2 + 2$, as we get

$$v^2 = 2f(s + n) = 2f's \quad \Rightarrow s = \frac{f\ n}{f' - f} \qquad ....(2)$$

Also for point B using the eqn $s = ut + \frac{1}{2}at^2$, we get

$$s = \frac{1}{2}f't^2$$

Substituting values of t and s from equations (1) and (2) in the above relation, we get

$$\frac{f\ n}{f' - f} = \frac{1}{2}f'\frac{f^2m^2}{(f' - f)^2} \Rightarrow (f' - f)n = \frac{1}{2}ff'm^2$$

27. (b) Let A and B be displaced by a distance x then Change in moment of $(A + B)$ = applied moments

$$\Rightarrow (A + B) \times x = H \Rightarrow x = \frac{H}{A + B}$$

28. (b) We know that for positive real numbers $x_1, x_2, ..., x_n$, A.M. of k^{th} powers of $x'_i s \geq k^{th}$ the power of A.M. of $x'_i s$

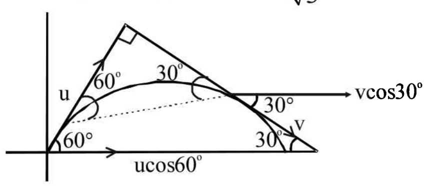
$$\Rightarrow \frac{\sum x_1^2}{n} \geq \left(\frac{\sum x_1}{n}\right)^2 \Rightarrow \frac{400}{n} \geq \left(\frac{80}{n}\right)^2$$

$\Rightarrow n \geq 16$. So only possible value for $n = 18$

29. (d) $u \cos 60° = v \cos 30°$

(as horizontal component of velocity remains the same)

$$\Rightarrow u \cdot \frac{1}{2} = v \cdot \frac{\sqrt{3}}{2} \text{ or } v = \frac{1}{\sqrt{3}}u$$

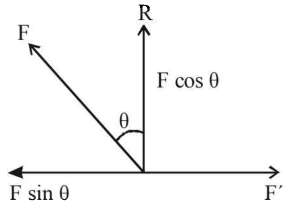


30. **(d)** Let F be the larger force

Given $R = \frac{F}{3}$

Resolving F in horizontal and vertical direction

$$R = F\cos\theta \Rightarrow \cos\theta = \frac{1}{3}$$



$$F' = F\sin\theta = F \times \frac{2\sqrt{2}}{3}$$

$\therefore\ F : F' = 3 : 2\sqrt{2}$

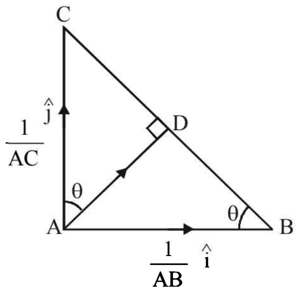
31. **(d)** If we consider unit vectors $\hat{i}$ and $\hat{j}$ in the direction AB and AC respectively, then as per quesiton, forces along AB and AC respectively are

$\left(\frac{1}{AB}\right)\hat{i}$ and $\left(\frac{1}{AC}\right)\hat{j}$

$\therefore$ Their resultant along $AD = \left(\frac{1}{AB}\right)i + \left(\frac{1}{AC}\right)j$

$\therefore$ Magnitude of resultant is

$$= \sqrt{\left(\frac{1}{AB}\right)^2 + \left(\frac{1}{AC}\right)^2} = \sqrt{\frac{AC^2 + AB^2}{AB^2 + AC^2}} = \frac{BC}{AB.AC}$$



But from figure $\Delta ABC \sim \Delta DBA$

$$\Rightarrow \frac{BC}{AB} = \frac{AC}{AD} \Rightarrow \frac{BC}{AB \times AC} = \frac{1}{AD}$$

$\therefore$ The required magnitude of resultant becomes $\frac{1}{AD}$.

32. **(b)** Clearly $(x, x) \in R \forall x \in W$. So R is relexive.

Let $(x, y) \in R$, then $(y, x) \in R$ as x and y have at least one letter in common. So, R is symmetric.

But R is not transitive for example

Let x = INDIA, y = BOMBAY and z = JOKER

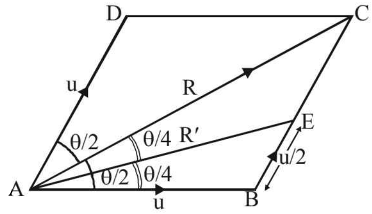
then $(x, y) \in R$ (A is common) and $(y, z) \in R$ (O is common) but $(x, z) \notin R$. (as no letter is common)

33. **(a)** $\sigma_x^2 = \frac{\sum d_i^2}{n}$ (Here deviations are taken from the mean).

Since A and B both have 100 consecutive integers, therefore both have same standard deviation and hence the variance.

$\therefore\ \frac{V_A}{V_B} = 1$ (As $\sum d_i^2$ is same in both the cases)

34. **(b)** For two velocities u and u at an angle θ to each other the resultant is given by



$$R^2 = u^2 + u^2 + 2u^2\cos\theta = 2u^2(1 + \cos\theta)$$

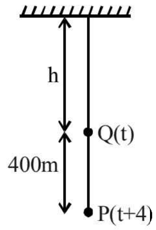
$$\Rightarrow R^2 = 4u^2\cos^2\theta/2 \text{ or } R = 2u\cos\frac{\theta}{2}$$

Now in second case, the new resultant AE (i.e., R') bisects $\angle CAB$, therefore using angle bisector theorem in ΔABC, we get

$$\frac{AB}{AC} = \frac{BE}{EC} \Rightarrow \frac{u}{R} = \frac{u/2}{u/2} \Rightarrow R = u \Rightarrow 2u\cos\frac{\theta}{2} = u$$

$$\Rightarrow \cos\frac{\theta}{2} = \frac{1}{2} = \cos 60° \Rightarrow \frac{\theta}{2} = 60° \text{ or } \theta = 120°$$

35. **(a)** Using $h = \frac{1}{2}gt^2$ and $h + 400 = \frac{1}{2}g(t+4)^2$



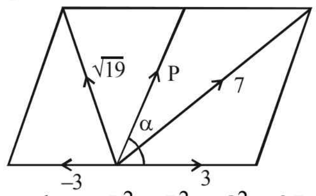
Subtracting, we get $400 = 8g + 4gt$

$\Rightarrow t = 8$ sec

$\therefore\ h = \frac{1}{2} \times 10 \times 64 = 320m$

$\therefore$ Desired height $= 320 + 400 = 720\ m$

36. **(c)** Given : Force $P = Pn$, $Q = 3n$, resultant $R = 7n$ & $P' = Pn$, $Q' = (-3)n$, $R' = \sqrt{19}\,n$



We know that $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

$\Rightarrow (7)^2 = P^2 + (3)^2 + 2 \times P \times 3\cos\alpha$

$\Rightarrow 49 = P^2 + 9 + 6P\cos\alpha$

$\Rightarrow 40 = P^2 + 6P\cos\alpha$ .....(i)

and $\left(\sqrt{19}\right)^2 = P^2 + (-3)^2 + 2P \times -3\cos\alpha$

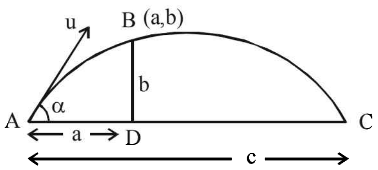
$\Rightarrow 19 = P^2 + 9 - 6P\cos\alpha$

$\Rightarrow 10 = P^2 - 6P\cos\alpha$ .....(ii)

Adding (i) and (ii) $50 = 2P^2$

$\Rightarrow P^2 = 25 \Rightarrow P = 5n$.

37. **(a)** Let B be the top of the wall whose coordinates will be (a, b). Range $(R) = c$



B lies on the trajectory

$$y = x\tan\alpha - \frac{1}{2}g\frac{x^2}{u^2\cos^2\alpha}$$

$$\Rightarrow b = a\tan\alpha - \frac{1}{2}g\frac{a^2}{u^2\cos^2\alpha}$$

$$\Rightarrow b = a\tan\alpha\left[1-\frac{ga}{2u^2\cos^2\alpha\tan\alpha}\right]$$

$$= a\tan\alpha\left[1-\frac{a}{\frac{2u^2}{g}\cos^2\alpha.\frac{\sin\alpha}{\cos\alpha}}\right]$$

$$= a\tan\alpha\left[1-\frac{a}{\frac{u^2.2\sin\alpha\cos\alpha}{g}}\right]$$

$$= a\tan\alpha\left[1-\frac{a}{\frac{u^2\sin2\alpha}{g}}\right]$$

$$= a\tan\alpha\left[1-\frac{a}{R}\right] \quad \left(\because R = \frac{u^2\sin^2\alpha}{g}\right)$$

$$\Rightarrow b = a\tan\alpha\left[1-\frac{a}{c}\right] \Rightarrow b = a\tan\alpha.\left(\frac{c-a}{c}\right)$$

$$\Rightarrow \tan\alpha = \frac{bc}{a(c-a)}$$

The angle of projection, $\alpha = \tan^{-1}\dfrac{bc}{a(c-a)}$

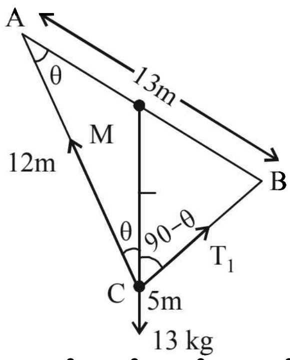
38. (a) Let the number of boys be x and that of girls be y.

$\Rightarrow 52x + 42y = 50(x+y) \Rightarrow 52x - 50x = 50y - 42y$

$\Rightarrow 2x = 8y \Rightarrow \dfrac{x}{y} = \dfrac{4}{1}$ and $\dfrac{x}{x+y} = \dfrac{4}{5}$

Required % of boys $= \dfrac{x}{x+y}\times100 = \dfrac{4}{5}\times100 = 80\,\%$

39. (a)



$\because 13^2 = 5^2 + 12^2 \Rightarrow AB^2 = AC^2 + BC^2$

$\Rightarrow \angle ACB = 90°$

Q m is mid point of the hypotenuse AB, therefore $MA = MB = MC \Rightarrow \angle A = \angle ACM = \theta$

Applying Lami's theorem at C, we get

$$\frac{T_1}{\sin(180-\theta)} = \frac{T_2}{\sin(90+\theta)} = \frac{13kg}{\sin90°}$$

$\Rightarrow T_1 = 13\sin\theta$ and $T_2 = 13\cos\theta$

$\Rightarrow T_1 = 13\times\dfrac{5}{13}$ and $T_2 = 13\times\dfrac{12}{13}$

$\Rightarrow T_1 = 5$ kg and $T_2 = 12$ kg

40. (d) Mean of $a, b, 8, 5, 10$ is 6

$\Rightarrow \dfrac{a+b+8+5+10}{5} = 10 \Rightarrow a+b = 7$ ...(i)

Variance of $a, b, 8, 5, 10$ is 6.80

$\Rightarrow \dfrac{(a-6)^2+(b-6)^2+(8-6)^2+(5-6)^2+(10-6)^2}{5} = 6.80$

$\Rightarrow a^2 - 12a + 36 + (1-a)^2 + 21 = 34$ [using eq. (i)]

$\Rightarrow 2a^2 - 14a + 24 = 0 \Rightarrow a^2 - 7a + 12 = 0$

$\Rightarrow a = 3$ or 4 $\Rightarrow b = 4$ or 3

$\therefore$ The possible values of a and b are $a = 3$ and $b = 4$ or, $a = 4$ and $b = 3$

41. (None)

p : x is an irrational number

q : y is a transcendental number

r : x is a rational number iff y is a transcendental number.

clearly r : $\sim p \leftrightarrow q$

Let us use truth table to check the equivalence of 'r' and 'q or p'; 'r' and $\sim(p \leftrightarrow \sim q)$

				1	2		3
p	q	~p	~q	~p ↔ q	q or p	p↔~q	~(p↔~q)
T	T	F	F	F	T	F	T
T	F	F	T	T	T	T	F
F	T	T	F	T	T	T	F
F	F	T	T	F	F	F	T

From columns (1), (2) and (3), we observe, none of the these statements are equivalent to each other.

$\therefore$ Statement 1 as well as statement 2 both are false.

$\therefore$ None of the options is correct.

42. (d) Let us make the truth table for the given statements, as follows :

p	q	p∨q	q→p	p →(q→p)	p →(p∨q)
T	T	T	T	T	T
T	F	T	T	T	T
F	T	T	F	T	T
F	F	F	T	T	T

From table we observe

$p \to (q \to p)$ is equivalent to $p \to (p \vee q)$

43. (b) The truth table for the logical statements, involved in statement 1, is as follows :

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

We observe the columns for $\sim(p \leftrightarrow \sim q)$ and $p \leftrightarrow q$ are identical, therefore

$\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$

But $\sim(p \leftrightarrow \sim q)$ is not a tautology as all entries in its column are not T.

$\therefore$ Statement-1 is true but statement-2 is false.

44. (c) For the numbers 2, 4, 6, 8, ..., $2n$

$$\bar{x} = \frac{2[n(n+1)]}{2n} = (n+1)$$

And $Var = \frac{\Sigma(x-\bar{x})^2}{2n} = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$= \frac{4\Sigma n^2}{n} - (n+1)^2 = \frac{4n(n+1)(2n+1)}{6n} - (n+1)^2$

$= \frac{2(2n+1)(n+1)}{3} - (n+1)^2 = (n+1)\left[\frac{4n+2-3n-3}{3}\right]$

$= \frac{(n+1)(n-1)}{3} = \frac{n^2-1}{3}$

$\therefore$ Statement-1 is false. Clearly, statement - 2 is true.

45. **(b)** Let $x \in A$ and $x \in B \Leftrightarrow x \in A \cup B$

$\Leftrightarrow x \in A \cup C \quad (\because A \cup B = A \cup C)$

$\Leftrightarrow x \in C$

$\therefore \quad B = C.$

Let $x \in A$ and $x \in B \Leftrightarrow x \in A \cap B$

$\Leftrightarrow x \in A \cap C \quad (\because A \cap B = A \cap C)$

$\Leftrightarrow x \in C \qquad \therefore B = C$

46. **(b)** Mean $= \frac{101 + d(1+2+3+......+100)}{101}$

$= 1 + \frac{d \times 100 \times 101}{101 \times 2} = 1 + 50d$

$\because$ Mean deviation from the mean = 255

$\Rightarrow \frac{1}{101}[|1-(1+50d)| + |(1+d)-(1+50d)| + |(1+2d)$
$-(1+50d)| + ... + |(1+100d)-(1+50d)|] = 255$

$\Rightarrow 2d[1+2+3+...+50] = 101 \times 255$

$\Rightarrow 2d \times \frac{50 \times 51}{2} = 101 \times 255 \Rightarrow d = \frac{101 \times 255}{50 \times 51} = 10.1$

47. **(b)** P : there is a rational number $x \in S$ such that $x > 0$

~P : Every rational number $x \in S$ satisfies $x \le 0$

48. **(b)** x Ry need not implies yRx

$\therefore$ R is not symmetric and hence not an equivalence relation.

$S : \frac{m}{n} s \frac{p}{q}$

Given $qm = pn \Rightarrow \frac{p}{q} = \frac{m}{n}$

$\therefore \frac{m}{n} s \frac{m}{n}$ (reflexive) $\frac{m}{n} s \frac{p}{q} \Rightarrow \frac{p}{q} s \frac{m}{n}$ (symmetric)

$\frac{m}{n} s \frac{p}{q}, \frac{p}{q} s \frac{r}{s} \Rightarrow qm = pn, ps = rq$

$\Rightarrow \frac{p}{q} = \frac{m}{n} = \frac{r}{s} \Rightarrow$ ms = rn (transitive).

S is an equivalence relation.

49. **(a)** $\sigma_x^2 = 4, \sigma_y^2 = 5, x = 2, y = 4$

$\frac{1}{5}\sum x_i^2 - (2)^2 = 4; \frac{1}{5}\sum y_i^2 - (4)^2 = 5$

$\sum x_i^2 = 40; \sum y_i^2 = 105 \Rightarrow \sum(x_i^2 + y_i^2) = 145$

$\Rightarrow \sum(x_i + y_i) = 5(2) + 5(4) = 30$

Variance of combined data

$= \frac{1}{10}\sum(x_i^2 + y_i^2) - \left(\frac{1}{10}\sum(x_i + y_i)\right)^2 = \frac{145}{10} - 9 = \frac{11}{2}$

50. **(b)** $x - y$ is an integer.

$\because x - x = 0$ is an integer $\Rightarrow A$ is reflexive.

Let $x - y$ is an integer

$\Rightarrow y - x$ is an integer

$\Rightarrow A$ is symmetric

Let $x - y, y - z$ are integers

$\Rightarrow x - y + y - z$ is also an integer

$\Rightarrow x - z$ is an integer

$\Rightarrow A$ is transitive

$\therefore A$ is an equivalence relation.

Hence statement 1 is true.

Also B can be considered as

xBy if $\frac{x}{y} = \alpha$, a rational number

$\because \frac{x}{x} = 1$ is a rational number

$\Rightarrow B$ is reflexive

But $\frac{x}{y} = \alpha$, a rational number need not imply $\frac{y}{x} = \frac{1}{\alpha}$, a rational number because

$\frac{0}{1}$ is rational $\Rightarrow \frac{1}{0}$ is not rational

$\therefore B$ is not an equivalence relation.

51. **(a)** Suman is brilliant and dishonest if and only if Suman is rich is expressed as

$Q \leftrightarrow (P \wedge \sim R)$

Negation of it will be $\sim (Q \leftrightarrow (P \wedge \sim R))$

52. **(b)** Median is the mean of 25th and 26th observation

$\therefore M = \frac{25a + 26a}{2} = 25.5a$

$M.D(M) = \frac{\Sigma|x_i - M|}{N}$

$\Rightarrow 50 = \frac{1}{50}[2 \times |a| \times (0.5 + 1.5 + 2.5 + ...24.5)]$

$\Rightarrow 2500 = 2|a| \times \frac{25}{2}(25) \Rightarrow |a| = 4$

53. **(a)** Let p : I become a teacher.

q : I will open a school

Negation of $p \to q$ is $\sim(p \to q) = p \wedge \sim q$

i.e. I will become a teacher and I will not open a school.

54. **(d)** **KEY CONCEPT :** If each observation is multiplied by k, mean gets multiplied by k and variance gets multiplied by k^2. Hence the new mean should be $2\bar{x}$ and new variance should be $k^2\sigma^2$.

So statement-1 is true and statement-2 is false.

55. **(b)** Let $X = \{1,2,3,4,5\}$

Total no. of elements = 5

Each element has 3 options. Either set Y or set Z or none. $(\because Y \cap Z = \phi)$

So, number of ordered pairs = 3^5

56. **(c)** Given

$n(A) = 2, n(B) = 4, n(A \times B) = 8$

Required number of subsets =

${}^8C_3 + {}^8C_4 + ... + {}^8C_8 = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$
$= 256 - 1 - 8 - 28 = 219$

57. (b) **Statement-2 :** $(p \to q) \leftrightarrow (\sim q \to \sim p)$
$\equiv (p \to q) \leftrightarrow (p \to q)$
which is always true.
So statement 2 is true
Statement-1 : $(p \wedge \sim q) \wedge (\sim p \wedge q) = p \wedge \sim q \wedge \sim p \wedge q$
$= p \wedge \sim p \wedge \sim q \wedge q = f \wedge f = f$
So statement-1 is true

58. (d) If initially all marks were x_i then $\sigma_1^2 = \dfrac{\sum_i (x_i - \bar{x})^2}{N}$

Now each is increased by 10

$$\sigma_1^2 = \frac{\sum_i [(x_i+10)-(\bar{x}+10)]^2}{N} = \frac{\sum_i (x_i - \bar{x})^2}{N} = \sigma_1^2$$

Hence, variance will not change even after the grace marks were given.

59. (b) $4^n - 3n - 1 = (1+3)^n - 3n - 1$
$= [{}^nC_0 + {}^nC_1.3 + {}^nC_2.3^2 + ... + {}^nC_n 3^n] - 3n - 1$
$= 9\,[{}^nC_2 + {}^nC_3.3 + ... + {}^nC_n 3^{n-2}]$
$\therefore$ $4^n - 3n - 1$ is a multiple of 9 for all n.
$\therefore$ $X = \{x : x$ is a multiple of $9\}$
Also, $Y = \{9(n-1) : n \in \mathbb{N}\} = \{$All multiples of $9\}$
Clearly $X \subset Y$. $\therefore X \cup Y = Y$

60. (d) First 50 even natural numbers are 2, 4, 6 ....., 100

$$\text{Variance} = \frac{\sum x_i^2}{N} - (\bar{x})^2$$

$$\Rightarrow \quad \sigma^2 = \frac{2^2 + 4^2 + ... + 100^2}{50} - \left(\frac{2+4+...+100}{50}\right)^2$$

$$= \frac{4(1^2 + 2^2 + 3^2 + ... + 50^2)}{50} - (51)^2$$

$$= 4\left(\frac{50 \times 51 \times 101}{50 \times 6}\right) - (51)^2 = 3434 - 2601$$

$\Rightarrow \quad \sigma^2 = 833$

61. (c)

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$
F	F	T	F	T
F	T	F	T	F
T	F	T	T	F
T	T	F	F	T

Clearly equivalent to $p \leftrightarrow q$

62. (c) $n(A) = 4, n(B) = 2 \Rightarrow n(A \times B) = 8$
The number of subsets of $A \times B$ having atleast
3 = elements $= {}^8C_3 + {}^8C_4 + ... + {}^8C_8$
$= 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$
$= 256 - 1 - 8 - 28 = 219$

63. (b) $\sim[\sim s \vee (\sim r \wedge s)]$
$= s \wedge \sim(\sim r \wedge s)$
$= s \wedge (r \vee \sim s)$
$= (s \wedge r) \vee (s \wedge \sim s)$
$= (s \wedge r) \vee 0$
$= s \wedge r$

64. (b) Sum of 16 observations $= 16 \times 16 = 256$
Sum of resultant 18 observations $= 256 - 16 + (3 + 4 + 5)$
$= 252$

Mean of observations $= \dfrac{252}{18} = 14$

65. (a) $f(x) + 2f\left(\dfrac{1}{x}\right) = 3x$ ....(1)

$f\left(\dfrac{1}{x}\right) + 2f(x) = \dfrac{3}{x}$ ....(2)

Adding (1) and (2) $\Rightarrow f(x) + f\left(\dfrac{1}{x}\right) = x + \dfrac{1}{x}$

Substracting (1) from (2) $\Rightarrow f(x) - f\left(\dfrac{1}{x}\right) = \dfrac{3}{x} - 3x$

On adding the above equations

$\Rightarrow f(x) = \dfrac{2}{x} - x$

$f(x) = f(-x) \Rightarrow \dfrac{2}{x} - x = \dfrac{-2}{x} + x \Rightarrow x = \dfrac{2}{x}$

$x^2 = 2$ or $x = \sqrt{2}, -\sqrt{2}$.

66. (a) $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$
$\Rightarrow \{(p \vee q) \wedge (\sim q \vee q)\} \vee (\sim p \wedge q)$
$\Rightarrow \{(p \vee q) \wedge T\} \vee (\sim p \wedge q)$
$\Rightarrow (p \vee q) \vee (\sim p \wedge q)$
$\Rightarrow \{(p \vee q) \vee \sim p\} \wedge (p \vee q \vee q)$
$\Rightarrow T \wedge (p \vee q)$
$\Rightarrow p \vee q$

67. (d) $\bar{x} = \dfrac{2+3+a+11}{4} = \dfrac{a}{4} + 4$

$$\sigma = \sqrt{\sum \frac{x_i^2}{n} - (\bar{x})^2}$$

$$\Rightarrow 3.5 = \sqrt{\frac{4+9+a^2+121}{4} - \left(\frac{a}{4}+4\right)^2}$$

$$\Rightarrow \frac{49}{4} = \frac{4(134+a^2) - (a^2+256+32a)}{16}$$

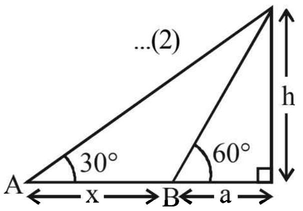
$\Rightarrow 3a^2 - 32a + 84 = 0$

68. (b) $\tan 30° = \dfrac{h}{x+a}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{x+a} \Rightarrow \sqrt{3}h = x + a$ ...(1)

$\tan 60° = \dfrac{h}{a} \Rightarrow \sqrt{3} = \dfrac{h}{a}$

$\Rightarrow h = \sqrt{3}a$ ...(2)



From (1) and (2)

$3a = x + a \Rightarrow x = 2a$

Here, the speed is uniform

So, time taken to cover x = 2 (time taken to cover a)

$\therefore$ Time taken to cover $a = \dfrac{10}{2}$ minutes = 5 minutes